4 - 10 / 2015

AMR

M.r

Complex Number

$x^2 - 9 = (x-3)(x+3)$

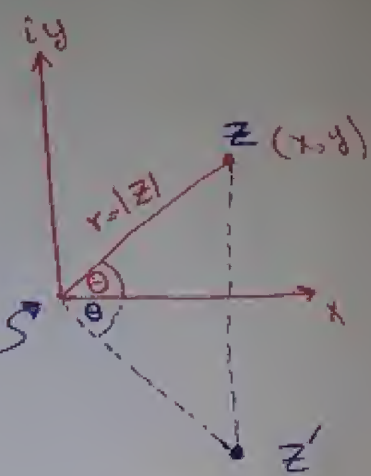
$x^2 + 9 = (x-3i)(x+3i)$

$$Z = x + iy = re^{i\theta} = r\left[\cos\theta + i\sin\theta\right]$$

$$\boxed{e^{i\ldots} = \cos\ldots + i\sin\ldots}$$

$Re(z) = x$

$Im(z) = y$



Conjugate المرافق

$\bar{Z} = x - iy = re^{-i\theta}$

$\overline{(w + z)} = \bar{w} + \bar{z}$

$\overline{(w z)} = \bar{w} \cdot \bar{z}$

$\overline{\left(\frac{w}{z}\right)} = \frac{\bar{w}}{\bar{z}}$

modulus المقياس

هي بعد النقطة من نقطة الأصل

$r = |z| = \sqrt{x^2 + y^2}$

[8] [Properities]

1. $|z_1 + z_2| \leq |z_1| + |z_2|$
2. $|z_1 + z_2| \geq |z_1| - |z_2|$
3. $|z_1 z_2| = |z_1| \cdot |z_2|$
4. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
5. $|z^n| = |z|^n$
6. $|z|^2 = z \cdot \bar{z}$

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argument السعة

$$\theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$$

هي الزاوية المحصورة بين النقطة ونقطة الأصل و ~~[illegible]~~ لمحور السينات.

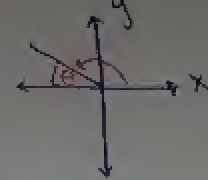
[Properities]

1. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
2. $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$
3. $\arg(z)^n = n \arg(z)$

(C)

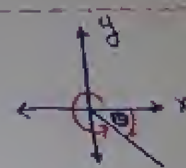
$y+$ ، $x+$

ربع أول $|\theta|$



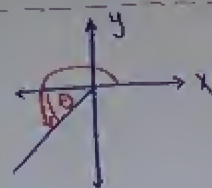
$y+$ ، $x-$

ربع ثاني $\pi - |\theta|$



$y-$ ، $x+$

ربع رابع $2\pi - |\theta|$



$y-$ ، $x-$

ربع ثالث $\pi + |\theta|$

adding & substracting.

$$Z_1 \pm Z_2 = (x_1 \pm x_2) + i\,(y_1 \pm y_2)$$

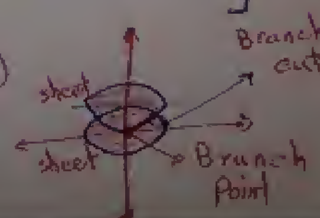
Roots of complex number

$$Z^{1/n} = r^{1/n}\left[\cos\left(\frac{\theta + 2\pi K}{n}\right) + i \sin\left(\frac{\theta + 2\pi K}{n}\right)\right]$$

$n = 2 \rightarrow$ جذرين

$K = (0, 1, 2, \ldots, n-1)$

$n \rightarrow$ تدل عدد الجذور



[d]

Sheet [1]

~~1- Find the real and imaginary~~

★ Find modulus & argument of .

(1) $z = \dfrac{2+i}{3+4i}$

$$|z| = \left|\frac{2+i}{3+4i}\right| = \frac{|2+i|}{|3+4i|} = \frac{\sqrt{4+1}}{\sqrt{9+16}} = \frac{1}{\sqrt{5}}$$

$$\arg(z) = \arg\left(\frac{2+i}{3+4i}\right) = \arg\underbrace{(2+i)}_{\text{ربع أول}} - \arg\underbrace{(3+4i)}_{\text{ربع أول}}$$

لأن نغير قيم الزوايا

$$\arg(z) = \tan^{-1}(1/2) - \tan^{-1}(4/3)$$

$$\arg(z) = \underline{-26.565}$$

(2) $\dfrac{1+2i}{3-4i} + \dfrac{2-i}{5i}$

$$|z| = \left|\frac{1+2i}{3-4i} + \frac{2-i}{5i}\right| = \left|\frac{5i(1+2i) + (3-4i)(2-i)}{(3-4i)(5i)}\right|$$

$$|z| = \left|\frac{5i - 10 + 6 - 3i - 8i - 4}{15i + 20}\right| = \left|\frac{-8-6i}{20+15i}\right|$$

$$|z| = \frac{|-8-6i|}{|20+15i|} = \frac{\sqrt{64+36}}{\sqrt{(20)^2+(15)^2}} = 0.4$$

$\boxed{E}$

$$\arg\left(\frac{-8-6i}{20+15i}\right) = \arg(-8-6i) - \arg(20+15i)$$

ربع ثالث   ربع أول

$$\arg(z) = \left(\pi + \tan^{-1}\left(\tfrac{6}{8}\right)\right) - \tan^{-1}\left(\tfrac{15}{20}\right) = \underline{\pi}$$

Find real & imagine part = Find polar form

$$z = x + iy = r\left[\cos\theta + i\sin\theta\right]$$

$\boxed{a}$ $(1+\sqrt{3}\,i)^6$

$$r = |z| = \left|(1+\sqrt{3}i)^6\right| = \left|(1+\sqrt{3}i)\right|^6$$

$$= \left(\sqrt{1+3}\right)^6 = \underline{2^6}$$

$$\theta = \arg(z) = \arg(1+\sqrt{3}i)^6 = 6\arg(1+\sqrt{3}i)$$

ربع أول

$$= 6 * \tan^{-1}\frac{\sqrt{3}}{1} = \underline{2\pi}$$

$$\therefore\ z = 2^6\left[\cos 2\pi + i\sin 2\pi\right] = 2^6\left[1 + i0\right]$$

$$z = \underbrace{2^6}_{\text{real part}} + \underbrace{0i}_{\text{Im part}}$$

5

(b) $\left(\frac{1+i}{1-i}\right)^4$

$r = |z| = \left|\frac{1+i}{1-i}\right|^4 = \left(\frac{|1+i|}{|1-i|}\right)^4 = \underline{1}$

$\theta = \arg(z) = r\arg\left(\frac{1+i}{1-i}\right)^4 = 4\left[r\arg(1+i) - \arg(1-i)\right]$

$\theta = 4\left[\tan^{-1}(1) - (2\pi - \tan^{-1}(1))\right] = \underline{-6\pi}$

$\therefore z = \cos(-6\pi) + i\sin(-6\pi)$

$z = 1 + 0i$

(2) show that

(a) $1 + \cos(\theta) + \cos(2\theta) + \dots \cos(n\theta)$

$= \frac{1}{2} + \frac{\sin\left((n+\frac{1}{2})\theta\right)}{2\sin(\theta/2)}$.

$e^{i\theta} = \cos\theta + i\sin\theta$

$\therefore \cos\theta = \mathrm{Re}(e^{i\theta})$

L.H.S $\longrightarrow \mathrm{Re}\left[1 + e^{i\theta} + e^{i2\theta} + \dots + e^{in\theta}\right]$

المتوالية الهندسية

$$a + ar + ar^2 + \cdots = a\,\frac{1-r^{n+1}}{1-r}$$

الحد الأول ← a ، الأساس ← r

$$\mathrm{Re}\left[\frac{1 - e^{i(n+1)\theta}}{1 - e^{i\theta}}\right] * \frac{e^{-i\frac{\theta}{2}}}{e^{-i\frac{\theta}{2}}}$$ بالضرب في

$$= \mathrm{Re}\left[\frac{e^{-i\theta/2} - e^{i(n+\frac{1}{2})\theta}}{e^{-i\theta/2} - e^{i\theta/2}}\right]$$

$$= \mathrm{Re}\left[\frac{\cos\left(\frac{-\theta}{2}\right) + i\sin\left(\frac{-\theta}{2}\right) - \cos\left(n+\frac{1}{2}\right)\theta - i\sin\left(n+\frac{1}{2}\right)\theta}{\cancel{\cos\left(\frac{-\theta}{2}\right)} + i\sin\left(\frac{-\theta}{2}\right) - \cancel{\cos\left(\frac{\theta}{2}\right)} - i\sin\left(\frac{\theta}{2}\right)}\right]$$

$$= \mathrm{Re}\left[\frac{\cos\left(\frac{\theta}{2}\right) - i\sin\left(\frac{\theta}{2}\right) - \cos\left(n+\frac{1}{2}\right)\theta - i\sin\left(n+\frac{1}{2}\right)\theta}{-2i\sin\left(\frac{\theta}{2}\right)}\right]$$

$$= \frac{1}{2} + \frac{\sin\left(n+\frac{1}{2}\right)\theta}{2\sin\left(\theta/2\right)} = R.H.S$$

13) show that

1b) $$\frac{(\cos 2\theta - i \sin 2\theta)^{7} (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 4\theta + i \sin 4\theta)^{12} (\cos 5\theta - i \sin 5\theta)^{-6}}$$

$$= \cos 107\theta - i \sin 107\theta$$

$$\text{L.H.S} = \frac{(e^{-i2\theta})^{7} (e^{i3\theta})^{-5}}{(e^{i4\theta})^{12} (e^{-i5\theta})^{-6}}$$

$$= \frac{e^{-i14\theta} \cdot e^{-i15\theta}}{e^{i48\theta} \cdot e^{i30\theta}} = e^{i\theta[-14-15-48-30]}$$

$$= e^{-i107\theta} = \cos(107\theta) - \sin(107\theta) = \text{R.H.S}$$

14) show that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$$

$\because |z|^2 = z \cdot \bar{z}$

$$\text{L.H.S} \longrightarrow (z_1 + z_2)(\overline{z_1 + z_2}) + (z_1 - z_2)(\overline{z_1 - z_2})$$

$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) + (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= z_1\bar{z}_1 + \cancel{z_1\bar{z}_2} + \cancel{z_2\bar{z}_1} + z_2\bar{z}_2 + z_1\bar{z}_1 - \cancel{z_1\bar{z}_2} - \cancel{z_2\bar{z}_1} + z_2\bar{z}_2$$

$$= 2z_1\bar{z}_1 + 2z_2\bar{z}_2 = 2|z_1|^2 + 2|z_2|^2 = \text{R.H.S}$$

5 Use De Moiver theorem to obtain $\cos 3\theta$ & $\frac{\sin 3\theta}{\sin\theta}$ in terms of power of $\cos\theta$. I

De moiver theorem

$$\boxed{(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)}$$

(wi)

0 → 1

1 → 1 1

2 → 1 2 1

3 → 1 3 3 1

Example $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

at $n = 3$

$$(\cos\theta + i\sin\theta)^3 = (\cos 3\theta + i\sin 3\theta)$$

$$= (\cos\theta)^3 + 3(\cos\theta)^2(i\sin\theta) + 3(\cos\theta)(i\sin\theta)^2 + (i\sin\theta)^3$$

$$= \cos^3\theta + 3i\cos^2\theta\sin\theta + -3\cos\theta\sin^2\theta - i\sin^3\theta$$

$$= \cos 3\theta + i\sin 3\theta$$

Real Part =

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\,[1 - \cos^2\theta]$$

Im Part

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$

$$\frac{\sin 3\theta}{\sin\theta} = 3\cos^2\theta - (1 - \cos^2\theta)$$